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A Dynamical Quasi-Boolean System

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Abstract

This paper provides the description of an original theoretical concept: the concept of Quasi-Boolean system. The concept can be used in any process that involves non-numerical values. The main idea is to express these types of values using float and Boolean values. The concept represents the theoretical fundament in the modeling of the online learning process based on the system theory.

Keywords: *Quasi-Boolean system, Boolean function, pseudo-Boolean function*

Introduction

In spite of the numerous, different computer-assisted instructional systems developed during the last few years, the studies in the field of e-learning have revealed problems caused by the holistic and complex features of the instructional process. The process of modeling real situations that are not governed by clear rules is a difficult one and implies new, unconventional approaches.

In this paper, there is presented a new concept, namely the Quasi-Boolean system, which was introduced by the author in [6]. The concept was used to model the instructional process, but its application area is not limited to that. The learning process is modeled starting with the description of an automatic regulation system. The intelligent instructional system introduced in [6] uses a combined regulation in order to minimize the effects of the perturbations of the learning process and to nullify the error (the difference between the reference value and the output value). "The input of the system represents the reference value, what the students have to realize after the learning (instructional) process has been finished." "The state (the system is sequential) is defined by the students' knowledge and skills." "The factors that affect the learning - motivation, goals, previous knowledge, interest, teaching styles, learning styles, classroom climate, parents, preoccupations, hobbies, etc - represent the perturbations." [6]

A largely description of the concept is presented in the work from the reference [7]. The concept was named a Quasi-Boolean system instead of pseudo-Boolean system because the equations of the systems are both pseudo-Boolean equations and Boolean equations. Pseudo-Boolean functions are "those set functions which are defined on a finite ground set and are given by closed algebraic formulae, and shall pay special attention to the case of multi-linear polynomial representations. Because of the analogy with Boolean functions, these functions will be called pseudo-Boolean." [1]

Such functions play a major role in the optimization problems, combinatorial theory, games theory, operations research, discrete mathematics, process modeling. Hammer has an important contribution in the study of these functions [2] [3] [4] [5]. A complete list of his publications can be found at the reference [8].

The concept of Quasi-Boolean system is a general and theoretical one, and, therefore, it can be used in any process, which interferes with real and Boolean numbers.

The Description of the Dynamical Quasi-Boolean Systems

In this chapter, we define the concept of a Quasi-Boolean system, which consists of pseudo-Boolean equations. The methods of solving Boolean and pseudo-Boolean equations and systems of equations can be found in a notable book of Hammer and Rudeanu [2].

The concept was set up on the idea that the output of the instructional process cannot be expressed using numerical values, but can be calculated using pseudo-Boolean formulas [2].

Definition 1. A discrete Boolean signal is a function according to the following scheme (Fig. 1):

$x : Z \to B_2, B_2 = \{0,1\}$ and Z is the set of integer numbers.

$\forall t \in Z, x(t) \in \{0,1\}$.

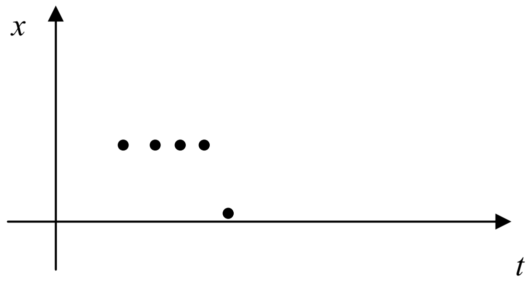

Fig. 1. An example of discret Boolean signal

Definition 2. A digital signal is a mapping of the following form:

$$x : Z \to M, M \subset Z .$$

Definition 3. A digital sequential signal is a mapping: $x : P \to M, M \subset Z$, where $P = \{1, 2, \ldots, n\}$ is a finite set of steps.

Definition 4. A Quasi-Boolean sequential signal is a map:

$x : P \to M, M \subset \Re \cup B_2$, where $P = \{1, 2, \ldots, n\}$ is a finite set of steps and $\Re$ is the set of real numbers.

Definition 5. Let's consider a vector $x = \left(x^1, x^2, \ldots, x^l, x^{l+1}, \ldots, x^n\right)$ with n elements, called Quasi-Boolean vector with $x^i \in \Re, i = \overline{1,l}$ and $x^i \in B_2, i = \overline{l+1,n}$.

We called the real part of vector x (noted with $R(x)$) and the Boolean part of vector x (noted with $B(x)$) using the following formulas:

$$R(x) = \left(x^1, x^2, \ldots, x^l\right),$$

$$B(x)=\left(x^{l+1},x^{l+2},\dots,x^{n}\right).$$

So, a Quasi-Boolean vector can be described according to the formula:

$$x=(R(x),B(x)).$$

Definition 6. A dynamical and sequential Quasi-Boolean system is a pair of functions (f,g) defined according to the following formula:

$$\begin{cases} x^i(k) \ = f_1^{\,i}(B(x(k-1)),B(u(k-1)),B(v(k-1)))+ \\ \qquad\qquad + \ f_2^{\,i}(R(x(k-1)),R(u(k-1)),R(v(k-1))), i=\overline{1,n_1} \\ x^i(k)=f_3^{\,i}(B(x(k-1)),B(u(k-1)),B(v(k-1))), i=\overline{n_1+1,n} \\ y^i(k)=g_1^{\,i}(\,B(x(k)),B(u(k)))+g_2^{\,i}(\,R(x(k)),R(u(k))), i=\overline{1,p_1} \\ y^i(k)=g_3^{\,i}(\,B(x(k)),B(u(k))),\ i=\overline{p_1+1,p} \end{cases},$$

where:

$f=(f_1,f_2,f_3),\quad g=(g_1,g_2,g_3),$

$x\in X=\Re^{n_1}\times B_2^{n_2}, n_1+n_2=n$ is the states' set,

$u\in U=\Re^{m_1}\times B^{m_2}\ \ , m_1+m_2=m$ are the inputs' signals,

$v\in V=\Re^{r_1}\times B_2^{r_2}, r_1+r_2=r$ is the perturbations' set,

$y\in Y=\Re^{p_1}\times B_2^{p_2}, p_1+p_2=p$ is the input signals' set,

$f_1^{\,i}, i=\overline{1,n_1}$ are pseudo-Boolean functions,

$f_2^{\,i}, i=\overline{1,n_1}$ are real functions,

$f_3^{\,i}, i=\overline{n_1+1,n}$ are Boolean functions,

$g_1^{\,i}, i=\overline{1,p_1}$ are pseudo-Boolean functions,

$g_2^{\,i}, i=\overline{1,p_1}$ are real functions,

$g_3^{\,i}, i=\overline{p_1+1,p}$ are Boolean functions,

$k\in P$ is a finite set of sequences and x,u,v are Quasi-Boolean signals.

Observations

The term of pseudo-Boolean equation is used because it is a common term in the Boolean algebra. The signals processed by a Quasi-Boolean system are vectors with float and Boolean numbers. The transition functions are pseudo-Boolean, float and Boolean functions. The reason of choosing these combinations of functions is that the real situation cannot be modeled using only float values. Any Boolean function can be represented using a pseudo-Boolean function. So, to simplify the model the functions $f_3^{\,i}$ and $g_3^{\,i}$ can be ignored and the equations of a Quasi-Boolean system can be described using the following formula:

$$\begin{cases} x^i(k) = f_1{}^i\left(B(x(k-1)), B(u(k-1)), B(v(k-1))\right) + \\ \qquad + f_2{}^i\left(R(x(k-1)), R(u(k-1)), R(v(k-1))\right), i = \overline{1,n} \\ y^i(k) = g_1{}^i\left(B(x(k)), B(u(k))\right) + g_2{}^i\left(R(x(k)), R(u(k))\right), i = \overline{1,p} \end{cases}.$$

Using the interpolation formula of the pseudo-Boolean functions, the equations of the system become:

$$\begin{cases} x^i(k) = \sum_{\gamma} c_\gamma x^{n_1+1}(k-1)^{\alpha_{n_1+1}}, \ldots, x^n(k-1)^{\alpha_n}, u^{m_1+1}(k-1)^{\alpha_{m_1+1}}, \ldots u^m(k-1)^{\alpha_m}, \\ v^{r_1+1}(k-1)^{\alpha_{r_1+1}}, \ldots, v^r(k-1)^{\alpha_r} + f_2^{\,i}\left(R(x(k-1)), R(u(k-1)), R(v(k-1))\right), \\ i = \overline{1,n},\ \gamma = \alpha_{n_1+1}, \ldots, \alpha_n, \alpha_{m_1+1}, \ldots, \alpha_m, \alpha_{r_1+1}, \cdots, \alpha_r, \\ c_\gamma = f_1^i\left(\alpha_{n_1+1}, \ldots, \alpha_n, \alpha_{m_1+1}, \ldots, \alpha_m, \alpha_{r_1+1}, \cdots, \alpha_r\right) \\ y^i(k) = \sum_{\delta} b_\delta x^{n_1+1}(k)^{\alpha_{n_1+1}}, \ldots, x^n(k)^{\alpha_n}, u^{m_1}(k)^{\alpha_{m_1+1}}, \ldots, u^m(k)^{\alpha_m} + \\ g_2{}^i\left(R(x(k)), R(u(k))\right), \\ i = \overline{1,p}, \delta = \alpha_{n_1+1}, \ldots, \alpha_n, \alpha_{m_1+1}, \ldots, \alpha_m, b_\delta = g_1^i\left(\alpha_{n_1+1}, \ldots, \alpha_n, \alpha_{m_1+1}, \ldots, \alpha_m\right) \end{cases}.$$

Observation

The equations of the systems can be described using linear polynomials that depend on the previous states, perturbations and inputs of the system.

The systems are classified according to the linearity of the functions

$\left(f = (f_1, f_2), g = (g_1, g_2)\right)$, in variables x, u, v, y.

A system is called linear sequential and Quasi-Boolean if all its functions are linear. If a least one function is non-linear, the system is called a non-linear sequential and Quasi-Boolean system.

The Description of a Linear Quasi-Boolean System

A linear Quasi-Boolean system is described using the following system of equations:

$$\begin{aligned} x^i(k) &= f_1{}^i\left(B(x(k-1)), B(u(k-1)), B(v(k-1))\right) + \\ &\quad + f_2{}^i\left(R(x(k-1)), R(u(k-1)), R(v(k-1))\right), i = \overline{1,n} \\ y^i(k) &= g_1{}^i\left(B(x(k-1)), B(u(k-1))\right) + g_2{}^i\left(R(x(k-1)), R(u(k-1))\right), i = \overline{1,p} \end{aligned}.$$

In order to simplify the problem, the perturbations are interpreted as external measures (inputs of the studied system). So, the equations of system become:

$$\begin{aligned} x^i(k) &= f_1^i\left(B(x(k-1)), B(u(k-1))\right) + f_2{}^i\left(R(x(k-1)), R(u(k-1))\right) \\ y^i(k) &= g_1{}^i\left(B(x(k-1)), B(u(k-1))\right) + g_2{}^i\left(R(x(k-1)), R(u(k-1))\right) \end{aligned}.$$

To simplify the notations, x_k and y_k are used instead of $x(k)$ and $y(k)$.

Any linear pseudo-Boolean equation can be written using the following notation:

$$c_1 z_1 + d_1 \overline{z_1} + \ldots + c_l z_l + d_l \overline{z_l} \ .$$

The state of the system is described using the following equation:

$$\begin{aligned} x^i{}_k &= a_{i1} x_{k-1}^{n_1+1} + a_{i1}^{'} \overline{x_{k-1}^{n_1+1}} + \ldots + a_{i,n-n_1} x_{k-1}^{n} + \\ &+ a_{i,n-n_1}^{'} \overline{x_{k-1}^{n}} + b_{i1} u_{k-1}^{m_1+1} + b_{i1}^{'} \overline{u_{k-1}^{m_1+1}} + \ldots + b_{i,m-m_1} u_{k-1}^{m} + \\ &+ b_{i,m-m_1}^{'} \overline{u_{k-1}^{m}} + a_{i1}'' x_{k-1}^{1} + \ldots + a_{in_1}'' x_{k-1}^{n_1} + \\ &+ b_{i1}'' u_{k-1}^{1} + \ldots + b_{im_1}'' u_{k-1}^{m_1},\ i = \overline{1,n}. \end{aligned}$$

The output of the system is described the following equation:

$$\begin{aligned} y^i{}_k &= c_{i1} x_k^{n_1+1} + c_{i1}^{'} \overline{x_k^{n_1+1}} + \ldots + c_{i,n-n_1} x_k^{n} + \\ &+ c_{i,n-n_1}^{'} \overline{x_k^{n}} + d_{i1} u_k^{m_1+1} + d_{i1}^{'} \overline{u_k^{m_1+1}} + \ldots + d_{i,m-m_1} u_k^{m} + \\ &+ d_{i,m-m_1}^{'} \overline{u_k^{m}} + c_{i1}'' x_k^{1} + \ldots + c_{in_1}'' x_k^{n_1} + \\ &+ d_{i1}'' u_k^{1} + \ldots + d_{im_1}'' u_k^{m_1},\ i = \overline{1,p}. \end{aligned}$$

The following notations are used:

$$A = \begin{pmatrix} a_{11} & \ldots & a_{1n-n_1} \\ \ldots & \ldots & \ldots \\ a_{n1} & \ldots & a_{nn-n_1} \end{pmatrix} \ A' = \begin{pmatrix} a'_{11} & \ldots & a'_{1n-n_1} \\ \ldots & \ldots & \ldots \\ a'_{n1} & \ldots & a'_{nn-n_1} \end{pmatrix} \ A'' = \begin{pmatrix} a''_{11} & \ldots & a''_{1n1} \\ \ldots & \ldots & \ldots \\ a''_{n1} & \ldots & a''_{nn1} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & \ldots & b_{1m-m_1} \\ \ldots & \ldots & \ldots \\ b_{n1} & \ldots & b_{nm-m_1} \end{pmatrix} \ B' = \begin{pmatrix} b'_{11} & \ldots & b'_{1m-m_1} \\ \ldots & \ldots & \ldots \\ b'_{n1} & \ldots & b'_{nm-m_1} \end{pmatrix} \ B'' = \begin{pmatrix} b''_{11} & \ldots & b''_{1m_1} \\ \ldots & \ldots & \ldots \\ b''_{n1} & \ldots & b''_{nm_1} \end{pmatrix}$$

$$R(x) = x_R = \begin{pmatrix} x_1 \\ \ldots \\ x_{n_1} \end{pmatrix},\ B(x) = x_B = \begin{pmatrix} x_{n1+1} \\ \ldots \\ x_n \end{pmatrix},\ x = \begin{pmatrix} R(x) \\ B(x) \end{pmatrix}$$

$$R(u) = u_R = \begin{pmatrix} u_1 \\ \ldots \\ u_{m_1} \end{pmatrix},\ B(u) = u_B = \begin{pmatrix} u_{m1+1} \\ \ldots \\ u_m \end{pmatrix},\ u = \begin{pmatrix} R(u) \\ B(u) \end{pmatrix}$$

$$R(y) = y_R = \begin{pmatrix} y_1 \\ \dots \\ y_{p_1} \end{pmatrix},\ B(y) = y_B = \begin{pmatrix} y_{p1+1} \\ \dots \\ y_p \end{pmatrix},\ y = \begin{pmatrix} R(y) \\ B(y) \end{pmatrix}$$

$$\overline{x_B} = \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix} - \begin{pmatrix} x_{n_1+1} \\ \dots \\ x_n \end{pmatrix} = I_{n-n_1} - x_B$$

$$\overline{u_B} = \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix} - \begin{pmatrix} u_{m_1+1} \\ \dots \\ x_m \end{pmatrix} = I_{m-m_1} - u_B$$

$$C = \begin{pmatrix} c_{11} & \dots & c_{1n-n_1} \\ \dots & \dots & \dots \\ c_{p1} & \dots & c_{pn-n_1} \end{pmatrix}\ C' = \begin{pmatrix} c'_{11} & \dots & c'_{1n-n_1} \\ \dots & \dots & \dots \\ c'_{p1} & \dots & c'_{pn-n_1} \end{pmatrix}\ C'' = \begin{pmatrix} c''_{11} & \dots & c''_{1n1} \\ \dots & \dots & \dots \\ c''_{p1} & \dots & c''_{pn_1} \end{pmatrix}$$

$$D = \begin{pmatrix} d_{11} & \dots & d_{1m-m_1} \\ \dots & \dots & \dots \\ d'_{p1} & \dots & d_{pm-m_1} \end{pmatrix}\ D' = \begin{pmatrix} d'_{11} & \dots & d'_{1m-m_1} \\ \dots & \dots & \dots \\ d'_{p1} & \dots & d'_{pm-m_1} \end{pmatrix}\ D'' = \begin{pmatrix} d''_{11} & \dots & d''_{1m_1} \\ \dots & \dots & \dots \\ d''_{p1} & \dots & d''_{pm_1} \end{pmatrix}$$

The equations of the state and the output of the system become:

$$x_k = Ax_{B\,k-1} + A'\overline{x_{B\,k-1}} + Bu_{B_{k-1}} + B'\overline{u_{B_{k-1}}} + A''x_{R_{k-1}} + B''u_{R_{k-1}}$$

$$x_k = (A - A')x_{B\,k-1} + (B - B')u_{B_{k-1}} + A'I_{n-n_1} + B'I_{m-m_1} + A''x_{R_{k-1}} + B''u_{R_{k-1}}$$

and

$$y_k = Cx_{B\,k} + C'\overline{x_{B\,k}} + Du_{B_k} + D'\overline{u_{B_k}} + C''x_{R_k} + D''u_{R_k}$$

$$y_k = (C - C')x_{B\,k} + (D - D')u_{B_k} + C'I_{n-n_1} + D'I_{m-m_1} + C''x_{R_k} + D''u_{R_k}.$$

The matrices $\mathrm{A}, \mathrm{A}', \mathrm{A}'', \mathrm{B}, \mathrm{B}', \mathrm{B}'', \mathrm{C}, \mathrm{C}', \mathrm{C}'', \mathrm{D}, \mathrm{D}', \mathrm{D}''$ are built as follows:

$$\mathrm{A} = \begin{pmatrix} 0 \ \dots \ 0 & \\ \dots \ \dots \ \dots & A \\ \underbrace{0 \ \cdots \ 0}_{n_1} & \end{pmatrix}\ \mathrm{A}' = \begin{pmatrix} 0 \ \dots \ 0 & \\ \dots \ \dots \ \dots & A' \\ \underbrace{0 \ \cdots \ 0}_{n_1} & \end{pmatrix}\ \mathrm{A}'' = \begin{pmatrix} & 0 \ \dots \ 0 \\ A'' & \dots \ \dots \ \dots \\ & \underbrace{0 \ \dots \ 0}_{n-n_1} \end{pmatrix}$$

$$\mathrm{B}=\begin{pmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots B \\ \underbrace{0 \quad \cdots \quad 0}_{m_1} \end{pmatrix}\ \mathrm{B}'=\begin{pmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots B' \\ \underbrace{0 \quad \cdots \quad 0}_{m_1} \end{pmatrix}\ \mathrm{B}''=\begin{pmatrix} & 0 & \dots & 0 \\ B''\dots & & \dots & \dots \\ & \underbrace{0 \quad \dots \quad 0}_{m-m_1} \end{pmatrix}$$

$$\mathrm{C}=\begin{pmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots C \\ \underbrace{0 \quad \cdots \quad 0}_{n_1} \end{pmatrix}\ \mathrm{C}'=\begin{pmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots C' \\ \underbrace{0 \quad \cdots \quad 0}_{n_1} \end{pmatrix}\ \mathrm{C}''=\begin{pmatrix} & 0 & \dots & 0 \\ C''\dots & & \dots & \dots \\ & \underbrace{0 \quad \dots \quad 0}_{n-n_1} \end{pmatrix}$$

$$\mathrm{D}=\begin{pmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots D \\ \underbrace{0 \quad \cdots \quad 0}_{m_1} \end{pmatrix}\ \mathrm{D}'=\begin{pmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots D' \\ \underbrace{0 \quad \cdots \quad 0}_{m_1} \end{pmatrix}\ \mathrm{D}''=\begin{pmatrix} & 0 & \dots & 0 \\ D''\dots & & \dots & \dots \\ & \underbrace{0 \quad \dots \quad 0}_{m-m_1} \end{pmatrix}$$

The state's equation becomes:

$$x_k = (\mathrm{A}-\mathrm{A}''+\mathrm{A}'')x_{k-1} + (\mathrm{B}-\mathrm{B}'+\mathrm{B}'')u_{k-1} + \mathrm{A}'I_n + \mathrm{B}'I_m$$

$$x_k = (\mathrm{A}-\mathrm{A}''+\mathrm{A}'')^k x_0 + \sum_{i=0}^{k-1}(\mathrm{A}-\mathrm{A}''+\mathrm{A}'')^{k-1-i}(\mathrm{B}-\mathrm{B}'+\mathrm{B}'')u_i + {}$$
$$+ \sum_{i=0}^{k-1}(\mathrm{A}-\mathrm{A}''+\mathrm{A}'')^i(\mathrm{A}'I_n + \mathrm{B}'I_m).$$

The output's equation becomes:

$$y_k = (\mathrm{C}-\mathrm{C}'+\mathrm{C}'')x_k + (\mathrm{D}-\mathrm{D}'+\mathrm{D}'')u_k + \mathrm{C}'I_n + \mathrm{D}'I_m .$$

In the systems' theory, the order law is defined as a dependence:

$$u = Fx + Gv,$$

where:

$x \in X = \Re^{n_1} \times B_2^{n_2}, n_1 + n_2 = n$ is the states' set,

$u \in U = \Re^{m_1} \times B^{m_2}\ , m_1 + m_2 = m$ is the input signals' set,

$v \in V = \Re^{r_1} \times B_2^{r_2}, r_1 + r_2 = r$ is the perturbations' set,

F, G are matrices with integer coefficients.

The order laws can be expressed using a pseudo-Boolean equations' system.

Observation

Any real vector $x=(x_1,x_2,\dots,x_n)\in\Re^n$ can be written as a Boolean vector using the following condition:

$$q=(q_1,q_2,\dots,q_n)\in\Re^n$$

$$x_i=\begin{cases}0, x_i\le q_i\\ 1, x_i>q_i\end{cases}.$$

The Description of a Non-Linear Quasi-Boolean System

In the description of a non-linear Quasi-Boolean system, more cases can be taken into account, according to the linearity of the functions.

A complex situation is defined by the case of non-linearity of the real functions and this situation has to be studied according to the type of functions.

A simplified case is the case of a system with linear real functions, described as follows:

$$\begin{aligned} x^i{}_{pas(k)} &= f_1{}^i\left(B\left(x_{pas(k-1)}\right), B\left(u_{pas(k-1)}\right)\right)+f_2{}^i\left(R\left(x_{pas(k-1)}\right), R\left(u_{pas(k-1)}\right)\right)\\ y^i{}_{pas(k)} &= g_1{}^i\left(B\left(x_{pas(k-1)}\right), B\left(u_{pas(k-1)}\right)\right)+g_2{}^i\left(R\left(x_{pas(k-1)}\right), R\left(u_{pas(k-1)}\right)\right)\end{aligned}.$$

Considering the transformation of a pseudo-Boolean function as a sum of polynomials, the system can be described using the formulas below:

$$x^i{}_k=a_{i1}z_k^1+\dots+a_{io}z_k^o+a''_{i1}x_{k-1}^1+\dots+a''_{in_1}x_{k-1}^{n_1}+b''_{i1}u_{k-1}^1+\dots+b''_{im_1}u_{k-1}^{m_1},\ i=\overline{1,n},$$

$$y^i{}_k=c_{i1}z_k^1+\dots+c_{io}z_k^o+c''_{i1}x_k^1+\dots+c''_{in_1}x_k^{n_1}+d''_{i1}u_k^1+\dots+d''_{im_1}u_k^{m_1},\ i=\overline{1,p}.$$

For homogeneity, we considered all polynomials having the forms:

$$\left(x_{k-1}^{n_1+1}\right)^{\alpha_{n_1+1}}\dots\left(x_{k-1}^{n}\right)^{\alpha_n}\left(u_{k-1}^{m_1+1}\right)^{\beta_{m_1+1}}\dots\left(u_{k-1}^{m}\right)^{\beta_m}.$$

The number of these polynomials is $2^{n+m-n_1-m_1}$. The symbol o represents the number $n+m-n1-m1$ and the polynomial was noted with $z^i, i=\overline{1,n+m-n_1-m_1}$.

Conclusions

The concept presented in this paper is a theoretical concept that represents a generalization of the mathematical equations describing the concept of a dynamic system. This concept is useful in any process of modeling situations that cannot be expressed using only real numbers. An example is that of modeling human behavior in the learning process.

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Sistem dinamic cvasi-boolean

Rezumat

În acest articol este prezentată descrierea unui concept teoretic original, conceptul de sistem Cvasi-Boolean. Conceptul poate fi utilizat în modelarea oricărui proces care implică valori ne-numerice. Ideea principală a conceptului este de a exprima aceste valori cu ajutorul valorilor reale şi booleene. Conceptul reprezintă fundamentul teoretic în modelarea procesului de învăţământ online bazat pe teoria sistemelor.